

# Correlation corrections to the thermodynamic properties of spin asymmetric QGP matter

Kausik Pal

*Department of Physics, Serampore College, Serampore 712201, India.*

We calculate the free energy, entropy and pressure of the Quark Gluon Plasma (QGP) at finite temperature and density with a given fraction of spin-up and spin-down quarks using a MIT bag model with corrections up to  $\mathcal{O}(g^4 \ln g^2)$ . The expressions for the specific heat and the spin susceptibility are derived in terms of Fermi momentum and temperature. The effects of interaction between the quarks on the properties of the QGP phase are also investigated. Within our phenomenological model, we estimate the transition temperature  $T_c$  by constructing the phase boundary between the hadronic phase and the QGP phase. Finally, we compute the equation of state of the QGP and its dependence on the temperature and the density.

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## I. INTRODUCTION

One of the active areas of high energy physics research is the study of thermodynamic properties of interacting hadronic matter in the extreme conditions of temperature and/or density for the last few decades. At such high temperature and/or density the hadrons are expected to dissolve into their more fundamental constituents *viz.* quarks and gluons, forming a new state of matter called Quark Gluon Plasma (QGP) [1–3]. The basic theory that describes the strong interaction in terms of quarks and gluons is known as Quantum Chromodynamics (QCD). It is known that QCD is asymptotically free *i.e.* strong interaction becomes weak for processes involving high transferred momenta [4–6]. This means that the quarks and gluons appear to be weakly coupled at very short distances. At large separations, the effective coupling becomes progressively stronger resulting in the phenomenon called the confinement of color charges *i.e.* at low density and temperature, the quarks and gluons remain confined in color singlet hadrons that constitute hadronic or nuclear matter [7–9].

However, when the density or the temperature are high enough, the quarks and gluons start to play dominant roles in determining the thermodynamic properties of the system. At extreme temperature and/or density one expects a transition of hadronic matter into a phase dominated by quarks and gluons, where color is deconfined and the interaction becomes screened [10–16]. Such a phase, where any length scale of interest is greater than the screening length of the interaction, is known as QGP. At vanishing chemical potential, lattice QCD suggests that this phase transition happens at a temperature around  $T_c \simeq 170 - 190$  MeV [17, 18]. At non-zero chemical potential, predictions on the order and exact location of the phase transition differ among the various lattice groups [19–23]. This is because lattice quantities are not properly defined in such a case so that different groups investigated with different additional approximations.

The possibility of creating high temperature QGP by colliding heavy ions in the laboratory and studying this phase of matter has been the goal of experiments at CERN SPS and at the Relativistic Heavy Ion Collider (RHIC) facility at Brookhaven National Laboratory (BNL) [24–26]. ALICE, ATLAS and CMS Collaborations at the Large Hadron Collider (LHC) have provided further impetus to these studies [27–29]. This experimental search of the QGP needs reliable theoretical estimates of various signals which depend on the pressure, entropy, deconfinement temperature and the equation of state (EOS) etc. [30–33].

In recent years, significant progress has been made to understand the behavior of QGP phase of matter leading to major advancement in the theoretical front addressing some of the subtle issues of the quasiparticles excitation in such an environment [30–36]. One of the major developments, in this context, has been the Hard Thermal Loop (HTL) approximated perturbation theory where these issues are handled in a systematic manner and meaningful results are obtained after performing suitable resummations [35, 37]. These apart, many calculations have been performed to study the high temperature transport phenomenon of QGP including calculations of various transport coefficients like viscosity, conductivity etc. [30, 38–40]. Several attempts have also been made to study the EOS for interacting QCD matter [31, 32, 34, 36, 41, 42]. For example, new lattice results for the equation of state of QCD with  $2 + 1$  dynamical flavors were obtained in [41, 42]. Ref.[36] deals with hybrid model in constructing a deconfining phase boundary between the hadron gas and the QGP and provides a realistic EOS for the strongly interacting matter. Furthermore, in [31, 32] the author have studied the thermodynamic properties of weakly interacting unpolarized QGP

matter by using a MIT bag model within one gluon exchange (OGE) interaction. Such investigations have also been performed in [34], where the calculations have been extended to spin polarized matter.

Being motivated by this series of works, we undertake the present investigation to study the thermodynamic behavior of weakly interacting spin asymmetric QGP matter including correlation corrections by using a MIT bag model with non-zero chemical potential. Due to asymptotic freedom, one may expect that the quarks and gluons interact weakly. Thus, the properties of QCD might be computable perturbatively [43]. So in our calculations it is assumed QCD coupling constant  $\alpha_c = \frac{g^2}{4\pi} < 1$ . It is to be mentioned, that for weak coupling constant  $g$ , perturbation theory can only be worked out to a finite order. In the strong coupling limit, the perturbation theory fails and one has to resort to lattice results. However, in the present work we assume the coupling to be weak enough for the series expansion in terms of  $\alpha_c$  to converge. Although the perturbative expansion converge very slowly, the approach makes predictions consistent, when comparison is possible, with lattice results which do not contain such an approximation [43]. In our model for the computation of the thermodynamic quantities, we require the knowledge of total energy density of spin polarized matter with the inclusion of bubble diagrams [44], like what one does for the calculation of the correlation energy for degenerate electron gas [45, 46]. Without such corrections, however, the calculations are known to remain incomplete as the higher order terms beyond the exchange diagrams are affected by infrared divergences due to the exchange of massless gluons [44]. This indicates the failure of the naive perturbation series. We know that this problem can be cured by summing a class of diagrams that makes the perturbation series convergent and receives logarithmic corrections [44–46]. In the present work, we calculate various thermodynamic quantities like energy density, pressure, entropy density with correlation corrections within the one gluon exchange model for non-zero chemical potential and show how these quantities can be expressed in terms of the spin polarization parameter  $\xi$  and the temperature  $T$ . We compare some of our results with the previous calculations, whenever possible. In addition, the present work is extended further to estimate the specific heat, spin susceptibility of QGP with a given fraction of spin-up and spin-down quarks. Within our model, we estimate the critical temperature  $T_c$  for the transition between the hadronic phase to the quark phase. Furthermore, we determine the equation of state of QGP and its dependence on the temperature and the density.

The plan of the article is as follows. In Sec. II, we calculate the various thermodynamic quantities with corrections up to  $\mathcal{O}(g^4 \ln g^2)$ . The results are also incorporated in this section. Sec. III. is devoted to the summary and discussions.

## II. THERMODYNAMIC PROPERTIES WITH CORRELATION

In this section we calculate the thermodynamic properties, like energy density ( $E$ ), pressure ( $\mathcal{P}$ ), entropy density ( $\mathcal{S}$ ), free energy ( $\mathcal{E}$ ), specific heat ( $C_v$ ), spin susceptibility ( $\chi$ ) for QGP matter with explicit spin dependent quarks by using a MIT bag model with fixed bag pressure  $\mathcal{B}$ . The MIT bag model considers free particles confined to a bounded region by the bag pressure. This pressure depends on the quark-quark interaction. For short, the bag constant ( $\mathcal{B}$ ) is the difference between the energy densities of non-interacting and interacting quarks [34, 47]. Within this model, for the calculation of energy density and other related quantities, we assume the QGP is composed of the light quarks only, i.e. the up and down quarks which interact weakly, and the gluons which are treated as almost free [31]. We consider the color symmetric forward scattering amplitude of two quarks around the Fermi surface by the one gluon exchange interaction. The direct term does not contribute as it involves the trace of single color matrices, which vanishes, while the leading contribution comes from the exchange term [48]. We are dealing with quasiparticles whose spins are all eigenstates of the spin along a given direction, viz.  $z$ . The quasiparticle interaction ( $f_{pp'}^{ss'}$ ) can be decomposed into two parts, spin may be either parallel ( $s = s'$ ) or antiparallel ( $s = -s'$ ) corresponding to spin nonflip ( $f_{pp'}^{\text{nf}}$ ) or spin flip ( $f_{pp'}^{\text{f}}$ ) scattering amplitudes [48, 49], such that

$$f_{pp'}^{ss'} = f_{pp'}^{\text{nf}} + \frac{1}{2} f_{pp'}^{\text{f}}, \quad (1)$$

where  $p$  and  $p'$  are the momentum of the quasiparticles. Here, the factor  $1/2$  is due to the equal scattering possibilities involving spin-up spin-down and spin-down spin-up quarks, i.e.  $f_{pp'}^{+-} = f_{pp'}^{-+}$ , where super-scripts denote the spin indices.

Since spin and momentum have no preferred direction, we take the average over the angles  $\theta_1$  and  $\theta_2$  corresponding to spins  $s$  and  $s'$ . The angular averaged interaction parameter is given by [48, 50]

$$f_{pp'}^{ss'} = \frac{g^2}{9pp'} \int \frac{d\Omega_1}{4\pi} \int \frac{d\Omega_2}{4\pi} \left[ 1 + (\hat{p} \cdot \hat{s})(\hat{p}' \cdot \hat{s}') \right], \quad (2)$$

where  $g$  is the coupling constant.

In our calculations, it is assumed that the nuclear matter is the initial state with equal neutron and proton densities i.e.  $n_n = n_p$ , so that  $n_u = n_d$ , which means the contributions of u and d quarks are equal. Since, we are dealing with ultra-relativistic massless quarks, all of the thermodynamic quantities, as we shall see, are obtained as a function of temperature ( $T$ ) and Fermi momentum ( $p_f$ ). Although the Fermi momentum is not a suitable experimental quantity, in fact it is better to calculate baryon density as a function of temperature and Fermi momentum. Since the baryon number of a quark is  $\frac{1}{3}$ , we have baryon density  $n_b = \frac{1}{3}n_q$  and the quark density  $n_q$  is given by

$$\begin{aligned} n_q &= \sum_{s=\pm} n_q^s = N_c N_f \sum_{s=\pm} \int \frac{d^3p}{(2\pi)^3} n_p^s(T) \\ &= \frac{1}{\pi^2} \sum_{s=\pm} \left[ p_f^s{}^3 + \pi^2 T^2 p_f^s \right]. \end{aligned} \quad (3)$$

Here,  $N_c = 3$  and  $N_f = 2$  are the color and flavor degeneracy factors, and  $n_p^s(T)$  is the Fermi distribution function. Using the above Eq.(3), the Fermi momentum as a function of  $n_b$  and  $T$  can be written as

$$\begin{aligned} p_f(n_b, T) &= \left\{ \frac{3}{4} \pi^2 n_b + \sqrt{\frac{\pi^6 T^6}{216} [(1 + \xi)^{1/3} + (1 - \xi)^{1/3}]^3 + \frac{9}{16} \pi^4 n_b^2} \right\}^{1/3} \\ &\quad + \left\{ \frac{3}{4} \pi^2 n_b - \sqrt{\frac{\pi^6 T^6}{216} [(1 + \xi)^{1/3} + (1 - \xi)^{1/3}]^3 + \frac{9}{16} \pi^4 n_b^2} \right\}^{1/3}. \end{aligned} \quad (4)$$

Here,  $\xi = (n_q^+ - n_q^-)/(n_q^+ + n_q^-)$  is the spin polarization parameter with  $0 \leq \xi \leq 1$ .  $n_q^+$  and  $n_q^-$  correspond to the densities of spin-up and spin-down quarks, respectively.

### A. Energy, pressure and entropy density of the QGP

To calculate the total energy density of the QGP, we need to compute the energy densities for both quark and gluon separately, as we treat gluons as non-interacting. The leading contributions to the energy density of quarks are given by three terms viz. kinetic, exchange and correlation energy densities, i.e.

$$E_q = E_{kin} + E_{ex} + E_{corr}. \quad (5)$$

The total kinetic energy density for spin-up and spin-down quarks, including the color and flavor degeneracy factors for quarks, is

$$E_{kin} = \frac{3}{(2\pi)^2} \left\{ p_f^4 [(1+\xi)^{4/3} + (1-\xi)^{4/3}] + 2\pi^2 T^2 p_f^2 [(1+\xi)^{2/3} + (1-\xi)^{2/3}] + \frac{14}{15} \pi^4 T^4 \right\}, \quad (6)$$

where,  $p_f$  is the Fermi momentum of the unpolarized matter ( $\xi = 0$ ).

For spin asymmetric quarks, the exchange energy density consists of two terms  $E_{ex} = E_{ex}^{nf} + E_{ex}^f$ , and can be determined by evaluating the following integrals

$$E_{ex}^{nf} = \frac{N_f N_c^2}{2} \sum_{s=\pm} \int \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} f_{pp'}^{nf} n_p^s(T) n_{p'}^s(T), \quad (7)$$

$$E_{ex}^f = N_f N_c^2 \int \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} f_{pp'}^f n_p^s(T) n_{p'}^s(T). \quad (8)$$

The analytical expression for the total exchange energy density is found to be

$$E_{ex} = \frac{g^2}{(2\pi)^4} \left\{ p_f^4 [(1+\xi)^{4/3} + (1-\xi)^{4/3} + 2(1-\xi^2)^{2/3}] + \frac{4}{3} \pi^2 T^2 p_f^2 [(1+\xi)^{2/3} + (1-\xi)^{2/3}] + \frac{4}{9} \pi^4 T^4 \right\}. \quad (9)$$

It might be noted that the kinetic and the exchange energy density at low temperature have been calculated in Ref.[50], but here, the relevant quantities have been derived by retaining higher-order terms in  $T$ .

The next higher order correction to the energy density beyond the exchange term is the correlation energy. The detailed calculation of correlation energy for spin polarized matter have been given in [44] which we quote here:

$$E_{corr} \simeq \frac{1}{(2\pi)^3} \frac{1}{2} \int_0^{\pi/2} \sin^2 \theta_E d\theta_E \left\{ \Pi_L^2 \left[ \ln \left( \frac{\Pi_L}{\varepsilon_f^2} \right) - \frac{1}{2} \right] + 2\Pi_T^2 \left[ \ln \left( \frac{\Pi_T}{\varepsilon_f^2} \right) - \frac{1}{2} \right] \right\}, \quad (10)$$

with  $\theta_E = \tan^{-1}(|k|/k_0)$  and  $K \equiv (k_0, 0, 0, |k|)$  is the gluon 4-momentum. Including the leading finite temperature corrections to the gluon self-energy, the relevant  $\Pi_L$  and  $\Pi_T$  are determined to be [44, 51, 52]

$$\Pi_L = \left[ \frac{N_f g^2}{4\pi^2} \sum_{s=\pm} p_f^{s^2} + \left( N_c + \frac{N_f}{2} \right) \frac{g^2 T^2}{3} \right] \sin^{-2} \theta_E (1 - \theta_E \cot \theta_E), \quad (11)$$

$$\Pi_T = \left[ \frac{N_f g^2}{4\pi^2} \sum_{s=\pm} p_f^{s^2} + \left( N_c + \frac{N_f}{2} \right) \frac{g^2 T^2}{3} \right] \cdot \frac{1}{2} [1 - \sin^{-2} \theta_E (1 - \theta_E \cot \theta_E)]. \quad (12)$$

These are then inserted in Eq.(10) and performing the  $\theta_E$  integration, we estimate  $E_{corr}$ . The leading  $g^4 \ln g^2$  order contribution is given by

$$E_{corr} = \frac{g^4 \ln g^2}{(2\pi)^6} \cdot \frac{1}{8} \left\{ p_f^4 [(1+\xi)^{4/3} + (1-\xi)^{4/3} + 2(1-\xi^2)^{2/3}] \right. \\ \left. + \frac{16}{3} \pi^2 T^2 p_f^2 [(1+\xi)^{2/3} + (1-\xi)^{2/3}] + \frac{64}{9} \pi^4 T^4 \right\}. \quad (13)$$

It is to be noted that the exchange energy and the correlation energy for various  $\xi$  are always positive for massless quarks [31, 48, 49]. Thus, the interaction between the quarks inside the bag is repulsive and it helps the interacting quarks and the gluons to go out from the bag very easily. Therefore a transition to a QGP phase is more favored than in a non-interacting case [31].

For the energy density of gluons we have

$$E_g = 16 \int \frac{d^3 k}{(2\pi)^3} \frac{k}{e^{k/T} - 1} \\ = \frac{8}{15} \pi^2 T^4, \quad (14)$$

where 16 is the degeneracy of gluons.

Since our system is ultra-relativistic, there is a simple relation between the pressure and the energy density:

$$\mathcal{P} = \frac{1}{3} E. \quad (15)$$

To obtain the total energy density of the QGP, the bag pressure ( $\mathcal{B}$ ) should be included in addition to the quark and gluon contributions [31],

$$E_{QGP} = E_q + E_g + \mathcal{B}. \quad (16)$$

The pressure of the system is determined to be

$$\mathcal{P}_{QGP} = \mathcal{P}_q + \mathcal{P}_g - \mathcal{B}. \quad (17)$$

Once the value of  $\mathcal{P}$  is determined, one can readily calculate the entropy density of the system by evaluating

$$\mathcal{S}_{QGP} = \frac{\partial}{\partial T} (\mathcal{P}_{QGP}). \quad (18)$$

The thermodynamic properties of the system can be obtained by using the Helmholtz free energy relation

$$\mathcal{E}_{QGP} = E_{QGP} - T\mathcal{S}_{QGP}. \quad (19)$$

For the numerical estimation of all these quantities, following Refs.[31, 32], we take  $\alpha_c = \frac{g^2}{4\pi} = 0.2$ , as the coupling constant of QCD and the bag pressure  $\mathcal{B} = 208 \text{ MeV fm}^{-3}$  for zero hadronic pressure.

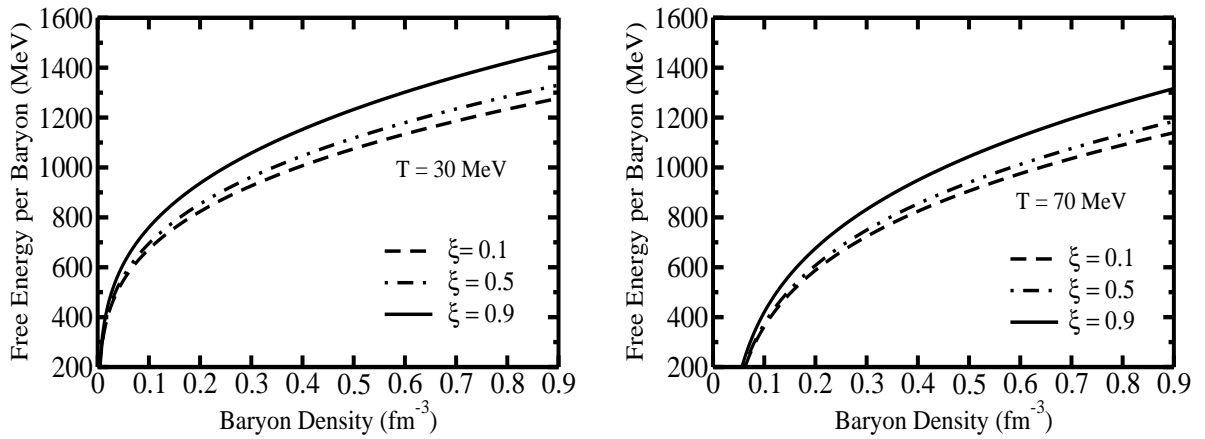


FIG. 1: Density dependence of the free energy per baryon at two different temperatures for different values of spin polarization  $\xi$ .

In Fig. 1, we plot the free energy per baryon as a function of baryon density for various order parameter  $\xi$  at two different temperatures, 30 MeV and 70 MeV. This shows that the free energy is larger with higher value of  $\xi$ . Therefore QGP is more stable when quarks are unpolarized.

Similarly, in Fig. 2, we show the temperature dependence of the free energy with various  $\xi$  at two different densities,  $0.2 \text{ fm}^{-3}$  and  $0.7 \text{ fm}^{-3}$  respectively. As expected the free energy decreases, as observed both in Fig. 1 and Fig. 2, with increasing temperature proving that QGP becomes more stable with unpolarized quarks.

In Fig. 3, the variations of entropy per baryon in terms of baryon density have been plotted with different values of order parameter  $\xi$ . The entropy per baryon is an important quantity connected to experimental observables of quark-gluon plasma formation in heavy



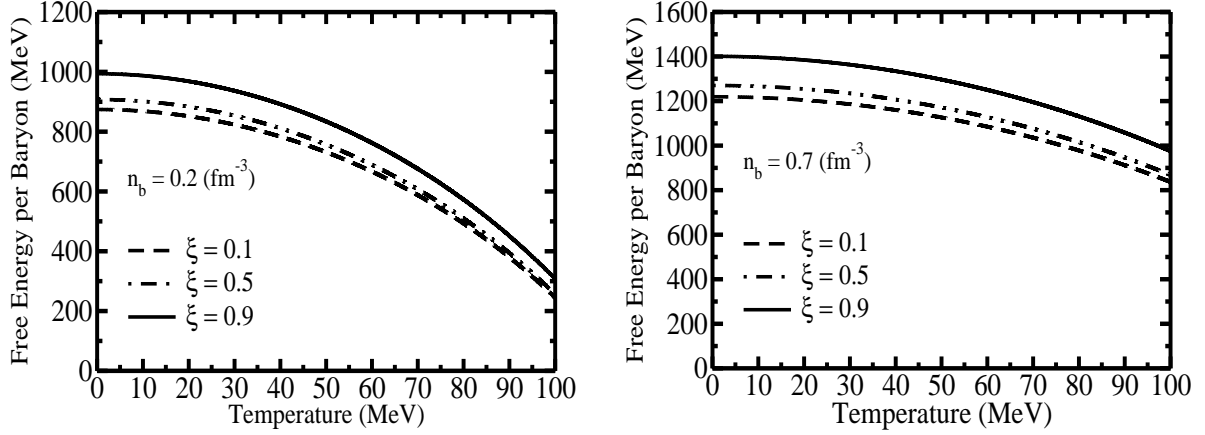


FIG. 2: Temperature dependence of the free energy per baryon at two different baryon densities for various  $\xi$ .

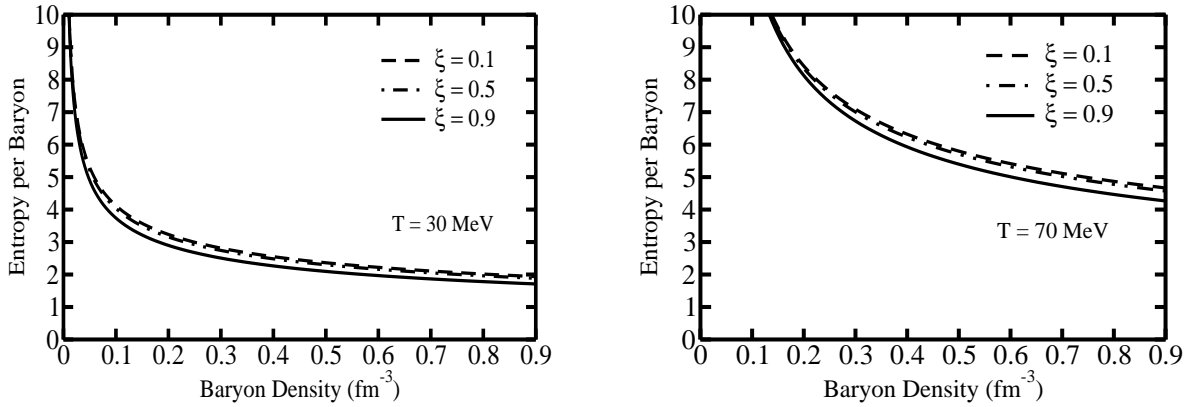


FIG. 3: Density dependence of the entropy per baryon at two different temperatures for various  $\xi$ .

ion collisions [53]. We find that the entropy decreases with increasing baryon density and at low density, entropy is approximately the same for all the values of the order parameter at a fixed temperature. Similarly, Fig. 4 shows the temperature dependence of the entropy per baryon for various  $\xi$ . This shows that the entropy per baryon in the QGP is an increasing function of temperature [31] and that the entropy decreases with increasing  $\xi$ . The numerical estimates suggest that the entropy per baryon is continuous along the phase boundary as observed in the right panel of Fig. 4, while for the left one the temperature reached in these computations is not high enough to make any conclusion. It should be mentioned that when the entropy per baryon varies continuously across the phase boundary, it is a smooth cross over.

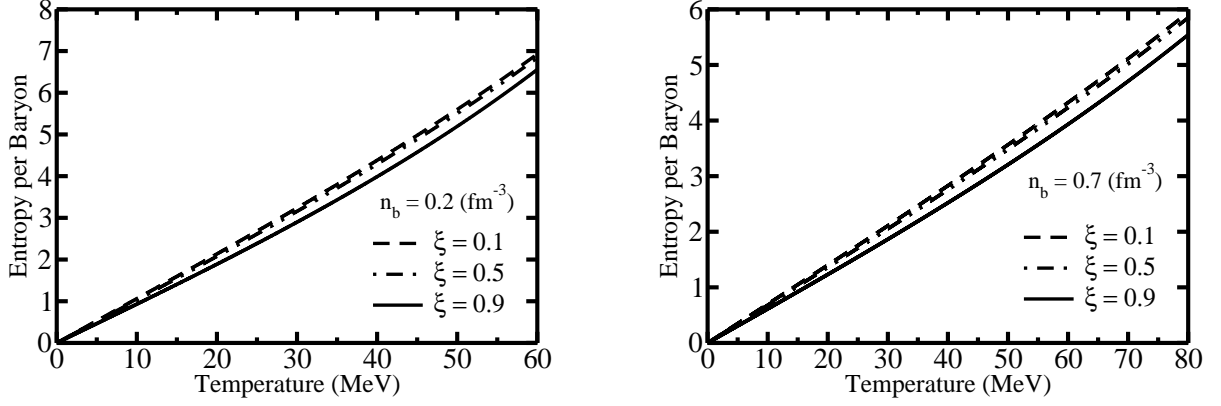


FIG. 4: Temperature dependence of the entropy per baryon at two different baryon densities for various  $\xi$ .

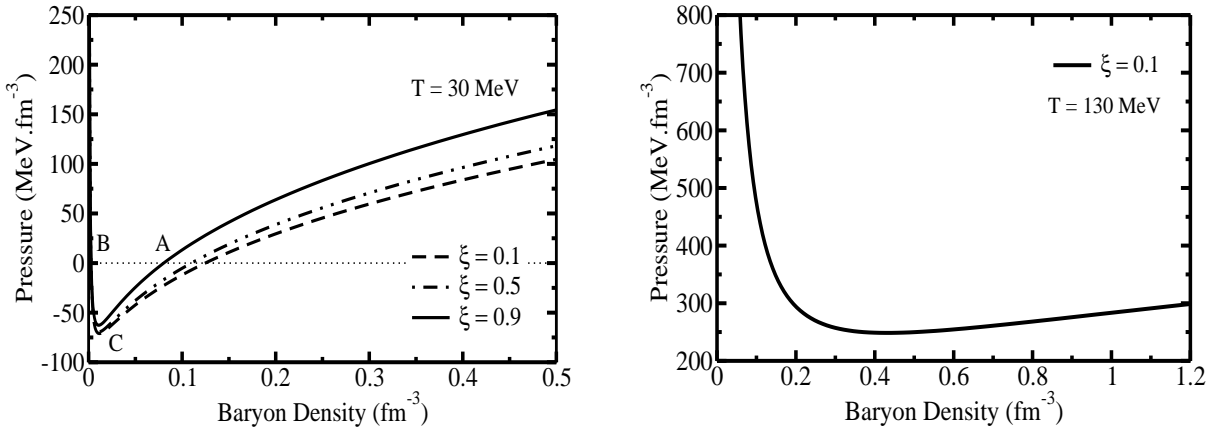


FIG. 5: The QGP equation of state as a function of baryon density for two different temperature with various  $\xi$ .

We have also studied the equation of state. The pressure is illustrated as a function of density in Fig. 5 with  $T = 30$  MeV (left panel) and  $T = 130$  MeV (right panel). The behavior observed for the pressure is driven by the fact that the interaction between the particles is attractive at large distance and repulsive at short distance. In the left panel of Fig. 5, the values of pressure and baryon density beyond the point ‘A’ correspond to the QGP phase while the segment BC correspond to the hadronic phase. The segment AC denotes the unstable state. In the right panel, we observe that the unstable state shows a tendency to disappear as the temperature increases. We also see, the pressure increases by increasing the

baryon density and for a fixed density the pressure of QGP is larger with polarized quark than the unpolarized one. Therefore, the transition to the deconfined phase with polarized quark at lower density would be favored in comparison to the one with unpolarized quarks.

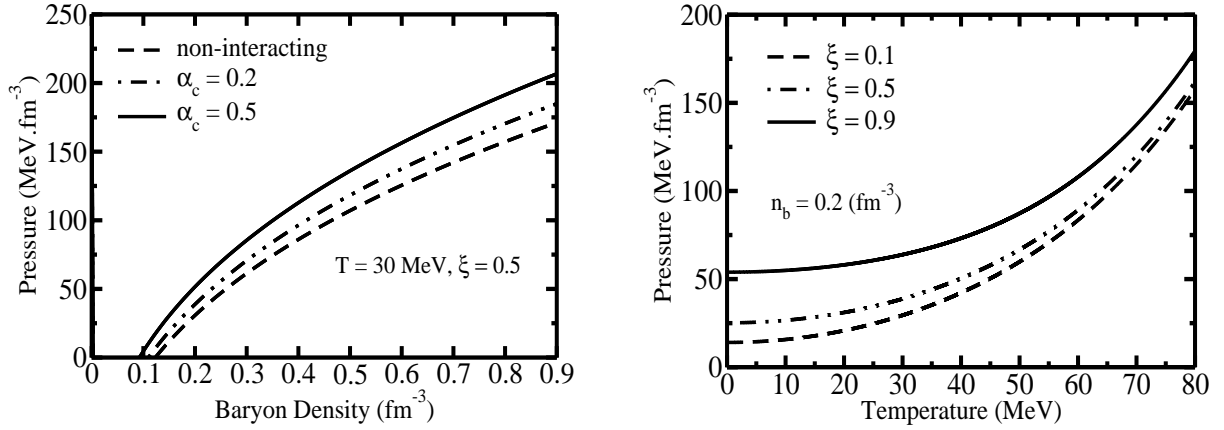


FIG. 6: The pressure for different coupling constants as a function of baryon density (left panel) and the pressure as a function of temperature for various  $\xi$  (right panel).

Similarly, in the left panel of Fig. 6, the variations of pressure of interacting QGP matter with baryon density for two different QCD coupling constants, and also for the noninteracting QGP at temperature 30 MeV has been shown. It is shown that at the same baryon density the pressure is larger for the interacting cases in comparison with the non-interacting one and if the interaction strength increases also the pressure raises further. Thus the interaction makes easier the quarks transition to the deconfined phase at lower density and an increase of the interaction strength pushes the pressure of quarks to be similar to the bag pressure at smaller baryon density. Therefore, the interaction of the QCD coupling constant reduces the value of the density to reach a transition [31].

In the right panel of Fig. 6, the pressure of spin polarized QGP matter as a function of temperature for different  $\xi$  has been plotted. We find that the pressure increases by increasing the temperature. It has been observed that at a constant temperature, the pressure of QGP is larger for polarized quarks than for unpolarized ones. These results indicate that the unpolarized state is energetically favorable for the QGP at any temperature and baryon density. The equation of state of spin asymmetric QGP matter becomes stiffer by increasing the baryon density (temperature). It has to be noted that the pattern is qualitatively similar

with that in Ref.[54].

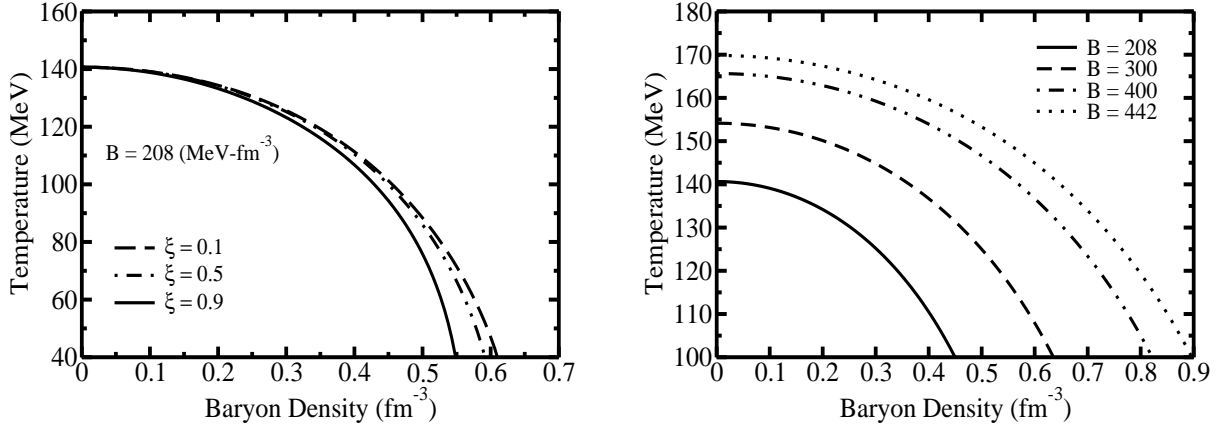


FIG. 7: The phase diagram with different  $\xi$  (left panel) and with different bag pressure (right panel).

The QGP phase diagrams are depicted in Fig. 7; in the left panel for different  $\xi$  at a constant bag pressure and in the right panel at different bag pressures for an unpolarized QGP. In the left panel, we see that the critical temperature is independent of the polarization parameter while critical density is different for unpolarized and polarized QGP. On the other hand, in the right panel it is shown that the critical temperature and density for the phase transition are different for different bag pressures and the critical values increase when increasing the bag pressure. This is because, as mentioned earlier, the interaction between the quarks is repulsive and helps the quarks to escape from the bags. This, in turn, causes the formation of QGP at smaller baryon densities and temperatures [31]. In the right panel of Fig. 7, it is shown that the critical temperature for the deconfined phase transition lies between  $130 < T_c < 170$  MeV. The exact location of the phase boundary varies as results are obtained for non-zero chemical potential. When the bag pressure is about  $442 \text{ MeV fm}^{-3}$ , the estimated results for critical temperature, i.e.  $T_c = 170$  MeV, is consistent with the lattice results [55–60]. It has to be mentioned that for our case, the critical parameters are determined from the phase boundary by using the condition that the bag pressure ( $\mathcal{B}$ ) is independent of chemical potential ( $\mu$ ) and temperature ( $T$ ), but in realistic cases  $\mathcal{B}$  may depend on both.

### B. Specific heat

The specific heat  $C_v$  at constant volume is defined as the quantity of energy needed to increase the temperature of a system by one unit, it reads as [61]

$$\begin{aligned}
 C_v &= \left( \frac{\partial E_{QGP}}{\partial T} \right)_v \\
 &= \left[ 1 + \frac{g^2}{18\pi^2} + \frac{g^4 \ln g^2}{144\pi^4} \right] \cdot \left[ (1 + \xi)^{2/3} + (1 - \xi)^{2/3} \right] \cdot 3p_f^2 T \\
 &\quad + \left[ \frac{74}{15} + \frac{g^2}{9\pi^2} + \frac{g^4 \ln g^2}{18\pi^4} \right] \cdot \pi^2 T^3.
 \end{aligned} \tag{20}$$

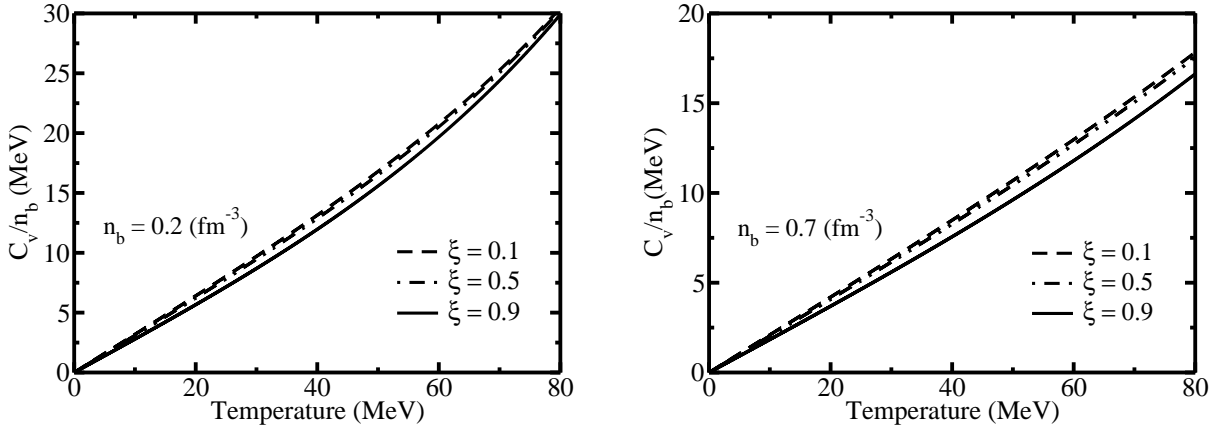


FIG. 8: The behavior of specific heat per baryon of QGP at two different baryon densities for various  $\xi$ .

The behavior of specific heat is plotted in Fig. 8 at two different densities. We see that the specific heat decreases by increasing the baryon density and polarization parameter. This is because the specific heat is a measure of energy fluctuation with temperature and the fluctuation is smaller when QGP is polarized. We also observe that the specific heat indicates complete continuity in its behavior.

### C. Spin susceptibility

The spin susceptibility can be determined by the change in energy of the system as quarks spin are polarized. In the small  $\xi$  limit, the energy density behaves like [50, 62–66]

$$E_{QGP}(\xi) = E_{QGP}(\xi = 0) + \frac{1}{2}\beta_s \xi^2 + \mathcal{O}(\xi^4), \tag{21}$$

where  $\beta_s$  is the spin stiffness constant. The spin susceptibility  $\chi$  is inversely proportional to the spin stiffness; mathematically  $\chi = 2\beta_s^{-1}$  [62]. Since only quarks are polarized and the energy density involves three leading terms, the spin susceptibility can be written as [50]

$$\chi^{-1} = \chi_{kin}^{-1} + \chi_{ex}^{-1} + \chi_{corr}^{-1}. \quad (22)$$

With the help of Eq.(21), each energy contribution to the susceptibility is

$$\begin{aligned} \chi_{kin}^{-1} &= \frac{p_f^4}{3\pi^2} \left( 1 - \frac{\pi^2 T^2}{p_f^2} \right), \\ \chi_{ex}^{-1} &= -\frac{g^2 p_f^4}{18\pi^4} \left( 1 - \frac{\pi^2 T^2}{3p_f^2} \right), \\ \chi_{corr}^{-1} &= -\frac{g^4 \ln g^2 p_f^4}{576\pi^6} \left( 1 + \frac{4\pi^2 T^2}{3p_f^2} \right). \end{aligned} \quad (23)$$

Using Eq.(22) and Eq.(23), the sum of all the contribution to the susceptibility is given by

$$\chi = \chi_P \left[ 1 - \frac{g^2}{6\pi^2} \left( 1 + \frac{4\pi^2 T^2}{3p_f^2} + \frac{\pi^4 T^4}{3p_f^4} \right) - \frac{g^4 \ln g^2}{192\pi^4} \left( 1 + \frac{7\pi^2 T^2}{3p_f^2} + \frac{4\pi^4 T^4}{3p_f^4} \right) \right]^{-1}. \quad (24)$$

where  $\chi_P$  is the Pauli susceptibility [50, 63]. The value of  $\chi$  can be estimated if  $g$ ,  $T$  and  $p_f$  are exactly known.

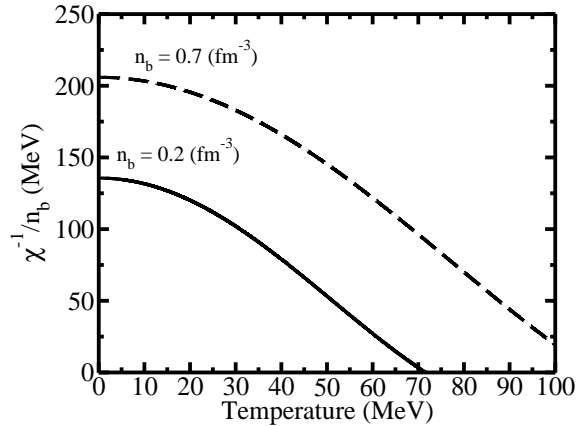


FIG. 9: The temperature dependence of inverse spin susceptibility.

The numerical estimation of  $\chi^{-1}$  per baryon is given in Fig. 9 for two different densities,  $0.2 \text{ fm}^{-3}$  and  $0.7 \text{ fm}^{-3}$ , respectively. We see susceptibilities blow up at different temperatures for two different densities. This can be identified as a physical instability of the QGP matter towards an ordered phase [63].

### III. SUMMARY AND DISCUSSIONS

In this work, we have investigated the thermodynamic properties of the QGP composed of the massless spin-up and spin-down quarks at finite temperature and density using a MIT bag model within one gluon exchange (OGE) interaction. Accordingly, we calculated the total free energy, entropy density, pressure as a function of baryon density and temperature for non-zero chemical potential of such a system up to  $\mathcal{O}(g^4 \ln g^2)$  that includes correction due to correlation effects. It was shown that the free energy increases by increasing the polarization parameter. This fact may suggest that the QGP with unpolarized quark is energetically favorable. We found that the entropy is a decreasing function of the density and an increasing function of the temperature. In the present phenomenological model, the equation of state (EOS) of the QGP with different order parameters has been computed. It was shown that the increase of temperature or density makes the EOS stiffer. We obtained a phase boundary for quark-hadron phase transition by making the bag constant independent of  $\mu$  and  $T$ . The values of critical parameters have been estimated for the transition from the hadronic phase to the QGP phase by using the EOS. It has been observed that the entropy per baryon is continuous along the phase boundary, indicating a cross over from the hadronic phase to the QGP phase. Moreover, we reported how the OGE interaction for the massless quarks affects the phase transition of the QGP and causes the system to reach the deconfined phase at smaller baryon densities and temperatures. In addition, the specific heat and the spin susceptibility of QGP have also been calculated and the specific heat decreases by increasing the baryon density in a continuous way. On the other hand, the spin susceptibility blows up at certain temperature for a definite baryon density. This may indicate the physical instability of the ordered phase of QGP.

It has to be mentioned that our results depend on the values of the bag pressure while a change in the value of the QCD coupling does not have any dramatic effect in our calculations. Moreover, an increase of the bag pressure might improve our results towards the lattice QCD calculations. Within the scope of the present model, the value of  $T_c$  obtained here, is close to the lattice QCD prediction and the inclusion of multi-gluon exchange processes may strengthen this conclusion. More works in this direction are therefore necessary to examine this issue, especially for multi-flavor systems. Leaving aside these questions, our studies might be meaningful for determining the approximate values of temperature, baryon

density, specific heat etc. regarding the signals of QGP and their detection for future studies in the Collider experiments.

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